Title: Quantum Approach for Multi-Objective Optimization Problems

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Description: The multi-objective unconstrained binary quadratic programming problem (mUBQP) is an interesting problem to study. Recently this problem got attention and a set of benchmark instances have been introduced also only recently [1]. Many multi-objective optimization problems can be converted to mUBQP, including multi-objective knapsack problem, multi-objective max-cut problem and bi-objective coloring problem. In general, multi-objective optimization is getting more and more attention, due to the complexity that systems-of-systems impose on modeling and optimization problems.

Due to the multi-objective nature of mUBQP, it is necessary to utilize multi-objective methodology to obtain a set of solutions that show the tradeoffs between the objectives. The UBQP is also known as QUBO (quadratic unconstrained binary optimization) problem and is the basis for solving optimization problems using quantum computers. Exact methods can solve UBQP/QUBO only with small or moderate-scale problem instances due to the NP-hard nature of UBQP. This leads to the first question to investigate:

Research question 1: Can quantum computing be beneficial for mUBQP problems?

In current classical solution methods, two implementation issues arise. The first one is the choice of the weight vector: how are various objectives weighted, to obtain Pareto optimal solutions. The second one is the choice of the scalarizing function: how are the various objectives function scaled, which affects the performance of algorithms greatly. In literature, multiple approaches and their pros and cons are known.

Research question 2: How does quantum computing affect the implementation issues and are the current approaches still valid?

In current quantum optimization practice, unconstrained problems are translated to QUBO/UBQP formulation using penalty values that have similarities to the Lagrange Multiplier approach of classical optimization [2]. Where theoretical values can be derived [3], those values do not perform well in quantum computing approaches, making the search for (the right) penalty values extremely difficult. However, this problem has great similarity to the aforementioned choice of the weight and scalarizing function in mUBQP.

Research question 3: Can the theory of choosing weight and scalarizing functions in classical mUBQP problems be used to find penalty values in QUBO/UBQP formulations?

Keywords: combinatorial optimization, quantum computing, complex systems

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