MMM Competition 2026

Problem 1: Security codes.

The organizers of the 30th edition of the MMM competition had to find a way to store the problem sheets safely. So they bought a new safe with buttons and discussed about the 4-digit security code. They know that used buttons wear down over time. This means that it will be visible which digits are used in the security code. The order and frequency of these digits is however unknown. In order to delay an attacker as much as possible, the objective is to find out how many distinct digits should be used to generate as much as possible feasible combinations. For example, if the code only uses the single digit 5, then the security code has to be 5555.

(a) Suppose the code uses two distinct digits, for example 1 and 5, how many feasible combinations are possible?

(b) Suppose the code uses three distinct digits, for example 1, 3 and 5, how many feasible combinations are possible?

(c) Suppose the code uses four distinct digits for example 1, 3, 5 and 8, how many feasible combinations are possible?

(d) How many distinct digits should the organizers use?

(e) How many distinct digits should the organizers use if the security code is a 5-digit code? Repeat the previous steps for a 5-digit code.

Problem 2: Sequences and sums.

Solve the following three questions.

(a) A number is called *seventeenish* if every two consecutive digits form a multiple of 17. For example: 517 is seventeenish because $51 = 3 \cdot 17$ and $17 = 1 \cdot 17$. How many numbers with two or more digits are seventeenish?

(b) Find all positive integers n consisting of four digits for which it holds that n plus the sum of the digits of n is equal to 3003.

(c) The following sum is equal to?

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{959}+\sqrt{960}}+\frac{1}{\sqrt{960}+\sqrt{961}}$$

Problem 3: The Drunken Jailor Strikes Back!

Several years ago there was an exercise about a drunken jailor who accidentally let some inmates escape from their cells. And now he's back! He guards a circular jail with 2000 cells numbered $1 - 2000$. There are 2026 inmates that need to be put in the cells. These inmates are also numbered $1 - 2026$. Our drunken jailor gets to decide which inmate goes in which cell. In his infinite wisdom he decides to do that according to the following procedure:

Inmate 1 is put in cell 1. Then, inmate 2 is put in cell 3, then inmate 3 is put in cell 6. This pattern continues, so cell number of inmate $n+1$ is exactly $n+1$ higher than the cell number of inmate n, where the cell directly after cell 2000 is not cell 2001, because there are only 2000 cells, but cell 1.

(a) In what cell does inmate 2026 end up?

(b) Since there are 2026 inmates and 'only' 2000 cells, there will be cells that have multiple inmates in them. What inmates, if any, eventually share a cell with inmate 2026?

Problem 4: Coin Flip Combinatorics.

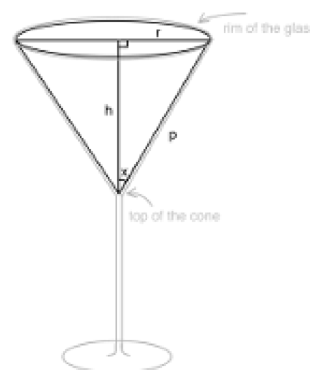
In a sequence of coin flips, one can keep a record of instances in which a tail is immediately followed by a head, a head is immediately followed by a head, etc. etc. We denote these subsequences by HT, HH, TT and TH. For example, in the sequence TTTHHTHTTTHHTTH of 15 coin flips we observe that there are two HH, three HT, four TH, and five TT subsequences.

How many different sequences of 21 coin flips will contain exactly two HH, five HT, six TH, and seven TT subsequences?

Problem 5: Cheers.

This year, Maastricht University celebrates its 50th anniversary. At the official party, drinks will be served in cocktail glasses with a conical shape. But what should the dimensions be? We denote the radius of the circular rim of a glass by r, the height of the cone by h, and the distance from the top of the cone to the rim by p. Half the angle at the top is denoted by x.

Questions:



(a) If the length p is kept fixed, which value of $\tan x$ maximizes the volume V of the glass? Note that a formula for V is provided by: $V = \frac{\pi \cdot h \cdot r^2}{3}$.

(b) If the length p is kept fixed, which value of $\tan x$ maximizes the ratio V/A, where V is the volume of the glass, and A the (inside) surface area of the cone? (Hint: first find a formula for A.)

(c) If the surface area A of the cone is kept fixed, which value of $\tan x$ maximizes the volume V of the glass?