MMM Competition 2026

Problem 1: Security codes.

The organizers of the 30th edition of the MMM competition had to find a way to store the problem sheets safely. So they bought a new safe with buttons and discussed about the 4-digit security code. They know that used buttons wear down over time. This means that it will be visible which digits are used in the security code. The order and frequency of these digits is however unknown. In order to delay an attacker as much as possible, the objective is to find out how many distinct digits should be used to generate as much as possible feasible combinations. For example, if the code only uses the single digit 5, then the security code has to be 5555.

(a) Suppose the code uses two distinct digits, for example 1 and 5, how many feasible combinations are possible?

(b) Suppose the code uses three distinct digits, for example 1, 3 and 5, how many feasible combinations are possible?

(c) Suppose the code uses four distinct digits for example 1, 3, 5 and 8, how many feasible combinations are possible?

(d) How many distinct digits should the organizers use?

(e) How many distinct digits should the organizers use if the security code is a 5-digit code? Repeat the previous steps for a 5-digit code.

Solutions.

(a) **14**: 1555, 5155, 5515, 5551, 1155,1515,1551,5115,5151,5511,1115,1151,1511,5111

(b) **36**: There are 12 options when one digit occurs double: 1135, 1153, 1315, 1351, 1513, 1531, 3115, 3151, 3511, 5113, 5131, 5311. So in total $12 \cdot 3 = 36$ combinations.

(c) **24**=4!

(d) **3**

(e) Denote by $T(n, k)$ the number of combinations for a n-digit code using k distinct digits. Then $T(n, k) = k \cdot (T(n-1, k) + T(n-1, k-1))$

with $T(n,1) = 1$, because there are k options for the first digit, then two cases apply to the remainder: (1) k occurs among the last $n-1$ digits, or (2) k does not occur among the last $n-1$ digits. Hence the recursive formula. This implies the following table:

n\k	1	2	3	4	5
1	1				
2	1	2			
3	1	6	6		
4	1	14	36	24	
5	1	30	150	**240**	120

Problem 2: Sequences and sums.

Solve the following three questions.

(a) A number is called *seventeenish* if every two consecutive digits form a multiple of 17. For example: 517 is seventeenish because $51 = 3 \cdot 17$ and $17 = 1 \cdot 17$. How many numbers with two or more digits are seventeenish?

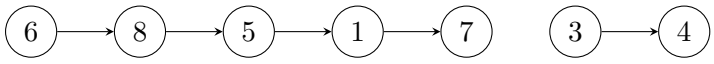
(b) Find all positive integers n consisting of four digits for which it holds that n plus the sum of the digits of n is equal to 3003.

(c) The following sum is equal to?

$$\frac{1}{\sqrt{1}+\sqrt{2}}+\frac{1}{\sqrt{2}+\sqrt{3}}+\frac{1}{\sqrt{3}+\sqrt{4}}+\ldots+\frac{1}{\sqrt{959}+\sqrt{960}}+\frac{1}{\sqrt{960}+\sqrt{961}}$$

Solutions.

(a) **11**: Multiples of 17 are: 17, 34, 51, 68, 85. This implies that a 1 should be followed by a 7, a 3 should be followed by a 4 etc. This yields the following successor graph:



Solutions: 17, 34, 51, 68, 85, 517, 685, 851, 6851, 8517, 68517.

(b) **2982 and 3000**: Write n as $abcd$. Then $1001a+101b+11c+2d = 3003$. Since $101b + 11c + 2d \leq 1026$, we know that $a = 2$ or $a = 3$. If $a = 2$, then since $11c + 2d \leq 117$, we need that $b = 9$. Now the remainder is 92, which has as a unique solution $c = 8$ and $d = 2$. If $a = 3$, then $b = c = d = 0$.

(c) **30**. Notice

$$\frac{1}{\sqrt{n}+\sqrt{n+1}} = \frac{\sqrt{n+1}-\sqrt{n}}{(\sqrt{n}+\sqrt{n+1})\cdot(\sqrt{n+1}-\sqrt{n})} = \frac{\sqrt{n+1}-\sqrt{n}}{n+1-n} = \sqrt{n+1}-\sqrt{n}$$

and so $\sqrt{2}-\sqrt{1}+\sqrt{3}-\sqrt{2}+\ldots+\sqrt{960}-\sqrt{959}+\sqrt{961}-\sqrt{960} = \sqrt{961}-\sqrt{1} = 31-1 = 30$

Problem 3: The Drunken Jailor Strikes Back!

Several years ago there was an exercise about a drunken jailor who accidentally let some inmates escape from their cells. And now he's back! He guards a circular jail with 2000 cells numbered $1 - 2000$. There are 2026 inmates that need to be put in the cells. These inmates are also numbered $1 - 2026$. Our drunken jailor gets to decide which inmate goes in which cell. In his infinite wisdom he decides to do that according to the following procedure:

Inmate 1 is put in cell 1. Then, inmate 2 is put in cell 3, then inmate 3 is put in cell 6. This pattern continues, so cell number of inmate $n+1$ is exactly $n+1$ higher than the cell number of inmate n, where the cell directly after cell 2000 is not cell 2001, because there are only 2000 cells, but cell 1.

(a) In what cell does inmate 2026 end up?

(b) Since there are 2026 inmates and 'only' 2000 cells, there will be cells that have multiple inmates in them. What inmates, if any, eventually share a cell with inmate 2026?

Solutions.

(a) Inmate 2026 gets in cell $1 + 2 + 3 + \ldots + 2026$. This sum is equal to $\frac{1}{2} \cdot 2026 \cdot 2027 = 2053351$ and that modulo 2000. Since $2053351 = 1026 \cdot 2000 + 1351$, inmate 2026 ends up in cell **1351**.

(b) This is an exercise in modulo arithmetics. Inmate n ends up in the same cell as inmate 2026 if $\frac{1}{2} \cdot 2026 \cdot 2027 \pmod{2000} = \frac{1}{2} \cdot n \cdot (n+1) \pmod{2000}$, or if $2026 \cdot 2027 - n \cdot (n+1) = 0 \pmod{2000}$, or $2026 \cdot 2027 - n \cdot (n+1)$ is divisible by 2000.

Notice the following three things:

1. $2000 = 2^4 \cdot 5^3$
2. $2026 \cdot 2027 - n \cdot (n+1) = (2026 - n)(2027 + n)$ and that one of these factors is even and one is odd.
3. $(2026 - n) + (2027 + n) = 4053$, which is not divisible by 5, so at most one of the two numbers is divisible by 5.

Combining (1) and (2) tells us that one of the two numbers (the even one) must be divisible by $2^4 = 16$. Combining (1) and (3) then tells us that the odd number must be divisible by $5^3 = 125$. This gives us two possibilities.

Possibility 1:

$$\begin{cases} 2026 - n \text{ divisible by } 16 & \text{gives} \quad n \equiv 10 \pmod{16} \\ 2027 + n \text{ divisible by } 125 & \text{gives} \quad n \equiv 27 \pmod{125} \end{cases}$$

We need to find the numbers n that satisfy both properties. So we check out all numbers n that are $27 \pmod{125}$ (i.e. $n = 27, 152, 277, 402, 527, 652, \ldots, 1902$) and we find that exactly one of them is equal to $10 \pmod{16}$, namely $n = 1402$. (Namely, $1402 = 11 \cdot 125 + 27$ and $1402 = 87 \cdot 16 + 10$). So inmate 1402 shares a cell with inmate 2026.

Possibility 2:

$$\begin{cases} 2026 - n \text{ divisible by } 125 & \text{gives} \quad n \equiv 26 \pmod{125} \\ 2027 + n \text{ divisible by } 16 & \text{gives} \quad n \equiv 11 \pmod{16} \end{cases}$$

Again we need to find the numbers n that satisfy both properties. So we check out all numbers n that are $26 \pmod{125}$ (i.e. n =26, 151, 276, 401, 526, 651,...,1902) and we find that exactly one of them is equal to $10 \pmod{17}$, namely $n = 651$. (Namely, $651 = 5 \cdot 125 + 26$ and $651 = 40 \cdot 16 + 11$). So inmate 651 also shares a cell with inmate 2026.

So the solution is that inmates **651 and 1402** share a cell with inmate 2026.

Problem 4: Coin Flip Combinatorics.

In a sequence of coin flips, one can keep a record of instances in which a tail is immediately followed by a head, a head is immediately followed by a head, etc. etc. We denote these subsequences by TH, HH, TT and TH. For example, in the sequence TTTHHTHTTTHHTTH of 15 coin flips we observe that there are two HH, three HT, four TH, and five TT subsequences.

How many different sequences of 21 coin flips will contain exactly two HH, five HT, six TH, and seven TT subsequences?

Solutions.

We have TH's and only five HT's. So we switch from T to H one more time than that we switch back from H to T. But that means that our sequence starts with a T and ends with an H. So the structure of the final order is

$$T_H_T_H_T_H_T_H_T_H_T_H_$$

where on the line segments after each T there might be some more T's and on the line segments after each H there might be some more H's.

The formula for selecting k elements out of a group of size n, where repetition is allowed (we are allowed to put multiple T's or H's on a single line segment) and order doesn't matter is

$$\binom{n-1+k}{k}$$

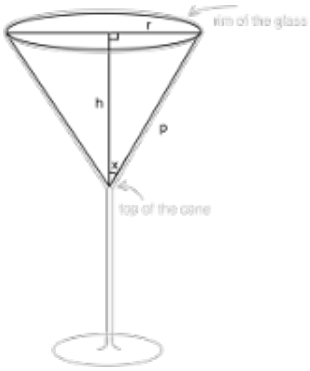
We use this formula to place seven more T's (since there were seven TT subsequences), giving us $n = 6$ and $k = 7$, so $\binom{12}{7} = 792$. Then we use the formula to place two more H's, giving $n = 6$ and $k = 2$. so $\binom{7}{2} = 21$. So the answer to the question is 792·21=**16632**.

Problem 5: Cheers.



This year, Maastricht University celebrates its 50th anniversary. At the official party, drinks will be served in cocktail glasses with a conical shape. But what should the dimensions be? We denote the radius of the circular rim of a glass by r, the height of the cone by h, and the distance from the top of the cone to the rim by p. Half the angle at the top is denoted by x.

Questions:



(a) If the length p is kept fixed, which value of $\tan x$ maximizes the volume V of the glass? Note that a formula for V is provided by: $V = \frac{\pi \cdot h \cdot r^2}{3}$.

(b) If the length p is kept fixed, which value of $\tan x$ maximizes the ratio V/A, where V is the volume of the glass, and A the (inside) surface area of the cone? (Hint: first find a formula for A.)

(c) If the surface area A of the cone is kept fixed, which value of $\tan x$ maximizes the volume V of the glass?

Solutions.

Before we start, here are some formulas that we will use throughout the solutions:

$$V = \frac{1}{3}\pi\, h\, r^2, \qquad A = \pi\, p\, r, \qquad h^2 + r^2 = p^2, \qquad \tan(x) = r/h.$$

The formula for the volume holds for the volume of every pyramid (regardless the precise shape of the base area) and therefore also for a cone: V = $\frac{1}{3}$ * height * base area.

The formula for the surface area of a cone (excluding the circular base) follows from the observation that you can fold a party hat (a cone) from a flat piece of paper: you need a disk with radius p from which a sector is missing. By comparing the circumferences of the rim of the glass ($2\pi r$) and the disk ($2\pi p$), you find the area as: $A = \frac{2\,\pi\,\mathrm{r}}{2\,\pi\,\mathrm{p}}\,\pi\, p^2 = \pi\, p\, r.$

The other two formulas are basic: Pythagoras' law and the definition of the tangent function.

(a) $V = \frac{1}{3}\pi\, h\, r^2 = \frac{1}{3}\pi\, h\, (p^2 - h^2)$, which is a function of the height *h*, for a fixed value of p. At $h = 0$ and at $h = p$, the volume is zero, but in between these values the volume is positive. It therefore has a maximum in the interval (0, p). Setting the derivative of V with respect to h to zero, gives: $V'(h) = \frac{\pi}{3}(p^2 - 3h^2) = 0$, from which it is obtained that there is a single extremum (hence a unique maximum), at $h = \frac{1}{\sqrt{3}}p$. For this value of *h*, it follows that $r = \frac{\sqrt{2}}{\sqrt{3}}p$, and therefore $\mathbf{\tan(x) = \sqrt{2}.}$

(b) In this question we focus on: $V/A = \frac{1}{3}\pi\, h\, (p^2 - h^2)/\pi\, p\, r = \frac{1}{3\,p}h\sqrt{p^2 - h^2}.$
Now, the derivative of V/A with respect to h, for fixed p, takes the form:
$\frac{1}{3\,p}\left(\sqrt{p^2 - h^2} + h\frac{-2\,h}{2\sqrt{p^2-h^2}}\right) = \frac{1}{3\,p\,\sqrt{p^2-h^2}}(p^2 - 2\,h^2)$. Setting it to zero gives a unique maximum at $h = \frac{1}{\sqrt{2}}p$. It follows that $r = \frac{1}{\sqrt{2}}p = h$, therefore $\mathbf{\tan(x) = 1}$. This implies that the top angle $2x$ equals π (rad), i.e., 90 degrees.

(c) This is the hardest question to solve. We must maximize $V = \frac{1}{3}\pi\, h\, r^2$, but under the constraint that $A = \pi\, p\, r$ remains fixed. Note that it is possible to express r in terms of h and A only: $A = \pi\, r\sqrt{r^2 + h^2}$ gives $A^2 = \pi^2\, r^2\, (r^2 + h^2)$, which is equivalent to $r^4 + h^2r^2 - \frac{A^2}{\pi^2} = 0$, from which r^2 (!) can be solved as:
$r^2 = -\frac{h^2}{2} \pm \frac{1}{2}\sqrt{h^4 + \frac{4A^2}{\pi^2}}$. Clearly only the positive solution is valid, from which we obtain: $V = \frac{\pi}{3}h\left(-\frac{h^2}{2} + \frac{1}{2}\sqrt{h^4 + \frac{4A^2}{\pi^2}}\right) = \frac{\pi}{6}\left(-h^3 + h\sqrt{h^4 + \frac{4A^2}{\pi^2}}\right).$
Its derivative with respect to *h*, for fixed A, is given by:
$$\frac{\pi}{6}\left(-3h^2 + \sqrt{h^4 + \frac{4A^2}{\pi^2}} + h\frac{4h^3}{2\sqrt{h^4+\frac{4A^2}{\pi^2}}}\right) = \frac{\pi}{6\sqrt{h^4+\frac{4A^2}{\pi^2}}}\left(-3h^2\sqrt{h^4 + \frac{4A^2}{\pi^2}} + 3h^4 + \frac{4A^2}{\pi^2}\right).$$

Setting it to zero, and rearranging it, produces: $3h^4 + \frac{4A^2}{\pi^2} = 3h^2\sqrt{h^4 + \frac{4A^2}{\pi^2}}$, which gives upon squaring: $9h^8 + 6h^4\frac{4A^2}{\pi^2} + (\frac{4A^2}{\pi^2})^2 = 9h^4(h^4 + \frac{4A^2}{\pi^2})$. Some terms cancel and we find $12h^4\frac{A^2}{\pi^2} = (\frac{4A^2}{\pi^2})^2$. Finally, $h^4 = \frac{4}{3}\frac{A^2}{\pi^2}$. Therefore: $h^2 = \frac{2}{\sqrt{3}}\frac{\mathrm{A}}{\pi}$.

For r^2 we find:

$r^2 = -\frac{h^2}{2} + \frac{1}{2}\sqrt{h^4 + \frac{4A^2}{\pi^2}} = -\frac{1}{\sqrt{3}}\frac{\mathrm{A}}{\pi} + \frac{1}{2}\sqrt{\frac{16A^2}{3\pi^2}} = -\frac{1}{\sqrt{3}}\frac{\mathrm{A}}{\pi} + \frac{2}{\sqrt{3}}\frac{\mathrm{A}}{\pi} = \frac{1}{\sqrt{3}}\frac{\mathrm{A}}{\pi} = \frac{1}{2}h^2.$

It means that the optimum is obtained for: $\mathbf{tan}(\boldsymbol{x}) = \frac{\boldsymbol{r}}{\boldsymbol{h}} = \frac{\mathbf{1}}{\sqrt{\mathbf{2}}} = \frac{\mathbf{1}}{\mathbf{2}}\sqrt{\mathbf{2}}$.