China Scholarships Council Project

PROJECT TITLE: Optimal strategies in visibility and patrolling problems

SUPERVISORY TEAM:

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PROJECT SUMMARY: There's an intruder in a guarded facility (such as a museum). The intruder decides where to hide, and simultaneously the guard decides where to look for the intruder. Both choose their respective locations in the facility, not knowing what the other will choose. The guard's objective is to see the intruder.

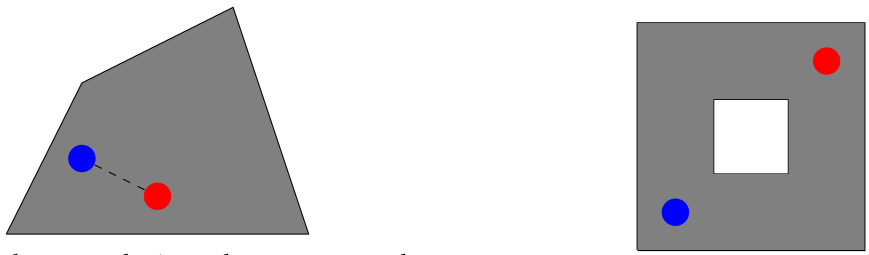
We depict the facility's ground plan as a body in the plane, the chosen location of the guard as a blue dot, and that of the intruder as a red dot. Now, if the facility is convex (panel a), the intruder would have no place to hide: she would be seen by the guard no matter where she, or the guard, choose to locate themselves. The problem becomes interesting if the facility is not convex. The guard is only able to see the intruder if the line segment connecting them lies entirely inside the facility.

What is the probability that the guard will be able to see the intruder?

We will be taking a game-theoretic approach to the problem. That is, we will be explicitly taking into account that both the intruder and the guard seek to optimize their respective strategies. A strategy for the intruder is a choice, possibly random, of a location in the facility. The same for the guard. Under this approach, the question above could be reformulated as: does the guard have a strategy such that she'll be able to see the intruder with a probability of, say, 95 percent, irrespective of the intruder's choice?

The above is but the most basic version of the visibility game; it is only the tip of the iceberg. As the project progresses, we will be studying several extensions of the problem, gradually bringing in more realism to our mathematical model. First, suppose that the guard is able to move thought the facility at a constant speed. Of course, eventually she will see the intruder; but how long does it take her to find him? In this extended setup the strategy for the guard consists of choosing a patrolling route through the facility, rather than just a single point. In a still more realistic (and challenging) scenario also the intruder is able to move. If he moves at a speed equal to that of the guard, the latter is not guaranteed to ever see the intruder. And what if the intruder's speed differs from the guard's?

The project touches upon several areas of mathematics and borders computer science. Visibility game could be seen as a game-theoretic analogue of the museum guarding problem, a textbook combinatorics problem. Probability theory naturally steps in as the optimal strategies of the players are randomized. The geometry of the facility is clearly crucial in determining the players' optimal strategies; visibility graphs will be called for to analyze and simplify the geometry. Where analytical solutions are not easily available, algorithmic solutions (e.g. for the optimal strategies) will be sought.



(a) The guard can see the intruder no matter where they are

(b) The guard is unable to see the intruder

KEYWORDS: Game theory, value, visibility graph, art gallery theorem, museum guarding problem.

REQUIREMENTS: A background in mathematics or a related field. The intrinsic motivation for deep research, and the interest and appreciation for theory are the most important qualifications.

KEY PUBLICATIONS OF THE SUPERVISORY TEAM:

Grigoriev, A., Van De Klundert, J. and Spieksma, F.C., 2006. Modeling and solving the periodic maintenance problem. *European Journal of Operational Research*, 172(3), pp.783-797.

Grigoriev, A., Sviridenko, M. and Uetz, M., 2007. Machine scheduling with resource dependent processing times. *Mathematical programming*, 110(1), pp.209-228.

Flesch, J. and Predtetchinski, A., 2016. Subgame-perfect ϵ-equilibria in perfect information games with common preferences at the limit. *Mathematics of Operations Research*, 41(4), pp.1208-1221.

Flesch, J. and Predtetchinski, A., 2017. A characterization of subgame-perfect equilibrium plays in Borel games of perfect information. *Mathematics of Operations Research*, 42(4), pp.1162-1179.

Cingiz, K., Flesch, J., Jean-Jacques, P. and Predtetchinski, A., 2020. Perfect information games where each player acts only once. *Economic Theory*, 69, pp.965-985.

Approved by the academic department (DAD): Alexandre Grigoriev